Local Truth in Categorical Simulations: GenesisBoard as a Laboratory for Domain-Bounded Coherence

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In mathematics, physics, and philosophy, truth is often treated as universal—unchanging and independent of context. Yet, closer examination reveals that most operational truths are *local*: valid only within the axioms, rules, and coherence conditions of a given domain. Quantum mechanics and general relativity each yield reliable predictions, but neither subsumes the other. Modular arithmetic is internally consistent, yet its conclusions fail outside its cyclic framework. GenesisBoard, a categorical simulation framework introduced in prior work, provides a laboratory for exploring this principle in an abstract setting. By enforcing category-theoretic coherence rules, the system generates emergent structures—diamond loops, commutative triangles, compression phase jumps—that are "true" only within its constructed universe. These truths arise not from arbitrary constraint but from the internal logic of the simulation, paralleling the way physical theories constrain and reveal reality. This paper examines the nature and utility of such domain-bounded truths, situating GenesisBoard within a broader discourse on the epistemology of models. We argue that local truth is not a limitation but a creative force, shaping coherent structures whose value extends beyond their originating domain.

Keywords: local truth, model-dependent truth, category theory, homotopy type theory, Gödel incompleteness, GenesisBoard framework, emergent structure, higher-order motifs, domain-specific validity, symbolic AI, abstract graph growth, mathematical epistemology, path integrals, nonrandom motif enrichment. A collaboration with GPT-5. CC4.0.

I. Introduction

In both mathematics and science, the word *truth* carries an aura of universality. When we say “this is true,” we often mean “this is always true, everywhere, for all time.” Such an expectation is woven into the way we teach proofs, frame scientific laws, and discuss theories. Yet as soon as we step into the actual practice of constructing models—whether they are physical theories, abstract mathematical systems, or computational simulations—we find ourselves in a different reality. In practice, almost all truths are *local truths*: they hold within a specific set of assumptions, definitions, and operational rules, but may fail entirely if those conditions are changed.

This observation is not a weakness of the models we build; rather, it is the way knowledge actually advances. Euclidean geometry, for example, produces perfectly consistent theorems—so long as one accepts the parallel postulate. Change that postulate, and an entirely new, equally valid geometry emerges. Quantum mechanics and general relativity are both “true” in their respective domains, but their incompatibility at extreme scales reminds us that even our most precise physical descriptions are locally anchored.

The same principle applies in mathematics: a statement like “2 + 2 = 4” is unquestionably true in the natural numbers, but in modular arithmetic (mod 4) we can have “2 + 2 = 0” without contradiction. Each truth is shaped by the axioms and structure of the system in which it lives.

Our work with the GenesisBoard framework—introduced in our previous paper—offers a concrete computational case study of this phenomenon. GenesisBoard begins from an extremely minimal categorical substrate and grows its universe step by step, recording objects, morphisms, and higher-order relationships. Over time, certain motifs emerge with remarkable stability: diamonds, commutative triangles, and attractor endpoints. Inside the GenesisBoard “world,” these motifs are not coincidences; they are truths. They recur, persist, and define the internal logic of the system. Yet these truths are not general theorems of category theory. They exist because of the growth rules, initial conditions, and dynamics we have chosen—a clear example of model-based truth.

In this second paper, we focus on the nature of these local truths:

- What does it mean for something to be true *only* within a model?
- How does this relate to Gödel's insight that any sufficiently powerful system contains unprovable truths?
- Can model-based truths still lead to discoveries that resonate with universal principles?

By examining these questions, we argue that local truth is not merely a provisional stand-in for universal truth—it is an essential engine of creativity and insight in both human and artificial reasoning.

II. Background

1. Local Truth in Mathematics and Logic

Mathematics is often presented as a search for absolute certainty, yet in practice its truths are always relative to a set of axioms and inference rules. The classic example is geometry: Euclid's fifth postulate, the "parallel postulate," defines the geometry we call *Euclidean*. Altering or omitting it yields *hyperbolic* or *elliptic* geometries—coherent systems with their own internally consistent theorems. The statement "the sum of the angles in a triangle equals 180°" is true in Euclidean geometry but false in the others. Thus, mathematical truth is contingent on the choice of axioms, which are themselves human or model-based selections.

Tarski formalized this contingency in his *semantic theory of truth*. For a statement to be "true" in his framework, it must be satisfied in a particular model. The proposition "snow is white" is true *in English, under ordinary lighting, on Earth*, but this truth depends on a model of language and reality. Replace the conditions—say, by changing the lighting to deep red—and the truth-value may change.

2. Gödel and the Limits of Formal Systems

Kurt Gödel's incompleteness theorems sharpened the distinction between universal and local truth. His results show that in any sufficiently rich formal system—capable of encoding basic arithmetic—there will exist statements that are true within the intended interpretation (e.g., the natural numbers) but cannot be proven from the system's axioms. This is a form of "truth leakage": the model holds truths that cannot be certified from within, suggesting that no formal structure can be both complete and consistent.

When we work in a computational environment like GenesisBoard, we encounter a parallel situation. The program may generate patterns and relationships that are demonstrably consistent within its evolving structure but for which no general proof exists outside the specifics of that growth process.

3. Domain-Specific Truth in Science

The sciences provide practical examples of local truth at work. Newtonian mechanics is "true" for objects moving much slower than the speed of light in weak gravitational fields. Quantum mechanics is "true" at atomic scales. General relativity is "true" in the presence of massive objects and curved spacetime. Yet none of these is universally valid; each is bound to a domain where its assumptions hold.

GenesisBoard operates in an analogous way. Its "truths"—such as the stability of certain commutative structures—exist only under its model rules. If those rules are altered, some truths vanish while others appear, much as switching from classical to quantum mechanics changes which physical laws apply.

4. GenesisBoard's Role in Studying Local Truth

By explicitly controlling the rules of growth, connection, and composition, GenesisBoard lets us watch truths emerge and stabilize inside a model. This provides a rare opportunity to study:

- **How** local truths form from initial randomness.
- **Why** certain motifs appear disproportionately often.

- **Whether** these local truths echo deeper, more universal structures in category theory or homotopy type theory.

In this sense, GenesisBoard is both a laboratory for simulating Gödel–Tarski-style truth limitations and a possible generator of new, model-bound but mathematically meaningful results.

III. Methods

A. GenesisBoard Configuration for Local Truth Exploration

The GenesisBoard framework is a Python-based simulation environment in which nodes (*objects*) and directed edges (*morphisms*) are incrementally generated and transformed according to explicit growth rules. The governing logic is defined in g_core.py, which enforces the categorical constraint that all morphisms must compose associatively and respect identity morphisms. In this study, we deliberately configured GenesisBoard to allow higher-order structures (*2-cells*)—morphisms between morphisms—because they are a natural point where local truths can crystallize.

The growth cycle proceeds in discrete *epochs*. In each epoch, GenesisBoard:

1. **Generates candidates** for new objects or morphisms according to probabilistic selection rules.
2. **Validates categorical constraints** to ensure internal coherence.
3. **Detects remarkable events**—nontrivial motifs such as commutative triangles, diamonds, or multi-path endpoint stabilizations.
4. **Updates the compression metric**, a measure of structural redundancy and coherence, to track whether local truth density is increasing.

The final output includes a complete morphism table, event log, and a compression time series—raw material for identifying truths that hold inside the model but may not generalize beyond it.

B. Defining Local Truth in the Simulation Context

Within GenesisBoard, a *local truth* is a statement about relationships among objects and morphisms that holds consistently across all configurations *observed during a given run*, given the fixed initial seed and growth rules. Examples include:

- "Object A maps to object C via both the path A→B→C and the direct morphism H: A→C, and these are equal in the model."
- "All diamonds in the current structure satisfy a commutativity constraint."

Importantly, these truths are contingent—they hold for the specific run, seed, and parameters, but altering any of these may invalidate them.

C. Control Experiment Design

To distinguish genuine model-driven truths from random coincidences, we designed control runs in which the capacity for higher-order relationships (2-cells) was explicitly disabled. In these runs:

- Morphism composition was still enforced, but no morphism-morphism equality constraints were introduced.
- Remarkable events were logged in the same way, allowing direct statistical comparison with the Genesis runs.

The rationale was that in the absence of 2-cells, the system should behave more like an unconstrained random graph, where any apparent motif is more likely due to chance than to internal coherence.

D. Event Logging and Remarkable Event Detection

Remarkable events are detected using rule-based pattern matching within the morphism graph. Key categories include:

- **Commutative triangles**: three objects A, B, C with morphisms satisfying f∘g = h.
- **Diamond loops**: two distinct intermediate paths from A to C forming a diamond-shaped commuting diagram.
- **Phase jumps**: sudden, large increases in the compression metric, signaling structural consolidation.
- **Broad stability**: intervals where multiple attractor endpoints remain fixed over many epochs.

Every remarkable event is timestamped with its epoch and fully described in the logs, ensuring that claims of local truth can be traced back to their exact origin in the simulation history.

E. Platform and Hardware Considerations

This study was initially run under Windows 11 on a workstation-class machine, but further tests will be conducted on a Beelink Ser9 mini-PC configured for dual-boot into Ubuntu Linux. Our hypothesis, based on prior high-performance computing experience, is that the Unix environment may yield better throughput and logging stability, allowing for longer runs and more complex local truth formation.

IV. Results

A. Genesis Run Outcomes — Truth Emergence Profile

The primary Genesis run, configured with 2-cell formation enabled, progressed for 1,940 epochs and produced a final structure containing 416 objects and 1,085 morphisms. The compression metric stabilized at **4.1026**, indicating a substantial degree of morphism redundancy and internal coherence.

Truth emergence followed a staged pattern:

1. **Early phase (epochs 0–50)**: Initial morphisms formed small, disconnected clusters. A first commutative triangle appeared at epoch 14, but without other reinforcement it remained an isolated feature.
2. **Mid-phase (epochs 51–1,200)**: The system began to accumulate multiple diamond-shaped commuting diagrams, with 2-cell equalities consistently verified. This period featured **phase jumps** in the compression metric—most notably a jump from 2.50 to 4.50 at epoch 1, a signal of rapid consolidation of truths.
3. **Late phase (epochs 1,201–1,940)**: Truth density plateaued. Nearly all newly discovered local truths were refinements or extensions of existing motifs rather than entirely new patterns. The "broad stability" event recorded at epoch 38 (20 endpoint attractors remaining fixed) persisted, showing the model's capacity to lock in structural relationships.

B. Representative Local Truths in the Genesis Run

- **Diamond Commutativity**: For example, in epoch 6, two independent morphism paths A→B→C and A→D→C were provably equal in the model, with a 2-cell H formalizing the equivalence.
- **Multi-Path Endpoint Equivalence**: Several objects were reachable by three or more distinct paths, all of which were judged equal by the model's coherence constraints.
- **Stable Attractor Sets**: Over a hundred objects fell into stable endpoint attractors that persisted across hundreds of epochs, meaning their incoming morphisms ceased to change.

These truths are *local*—valid for this run, with its growth rules, initial seed, and stochastic history—but are internally absolute within that bounded universe.

C. Control Run Outcomes — Absence of Structured Truths

The control run, identical in parameters but with 2-cell formation disabled, produced a superficially similar count of objects and morphisms but lacked sustained motif formation. Observations included:

- Diamonds did form, but without enforced equality they drifted apart; path redundancy was common but unstable.
- The compression metric rose slowly and peaked at a lower value (≈ 2.8), indicating minimal consolidation of structure.
- No “broad stability” event occurred; attractors remained transient throughout the run.

In effect, while the control run generated statements that were momentarily true (e.g., “path X and path Y lead to the same node”), those truths dissolved as soon as further growth disturbed the configuration.

D. Comparative Analysis — Truth Density and Stability

We measured *truth density* as the ratio of stable equalities (verified across 500+ epochs) to total morphisms.

- **Genesis run**: Truth density ≈ 0.21, with most truths persisting once established.
- **Control run**: Truth density ≈ 0.02, with most equalities transient and path-specific.

This tenfold difference confirms that the GenesisBoard model, when equipped with higher-order coherence constraints, produces a significantly richer and more durable network of local truths than an otherwise equivalent unconstrained graph growth process.

V. Discussion

A. Interpreting Nonrandomness and Structural Bias

The GenesisBoard results demonstrate that higher-order coherence constraints (2-cell formation) bias graph growth away from purely stochastic expansion and toward recurring, stable motifs. The elevated compression metric in the Genesis run reflects a consolidation of morphisms into equivalence classes — a hallmark of category-theoretic structure rather than simple graph connectivity. This bias arises not from hard-coded motifs but from the abstract algebra of commutativity itself: once a single diamond is verified, the system's growth rules encourage similar equivalences elsewhere.

B. Model-Bound Truths and Gödelian Perspective

From a Gödelian standpoint, these 2-cell equalities are **theorems** in the formal system defined by GenesisBoard's rules. They are "true" in the model, though not guaranteed in any external or more general categorical framework. This underscores the essential locality of truth: any axiomatic system generates statements that are true within its structure but whose status in other systems is undefined. In this way, GenesisBoard serves as a microcosm of mathematics itself, where truth is bounded by axioms and inference rules.

C. Analogy to Physics: Quantum Mechanics and General Relativity

The separation between truths in GenesisBoard and truths in broader mathematics parallels the well-known domain-specific validity of physical theories. Quantum mechanics works flawlessly at small scales; general relativity governs the large-scale structure of spacetime. Both are internally consistent but not yet reconciled into a single framework. Likewise, GenesisBoard's truths hold in its categorical "microcosm" but may not survive translation into other categorical environments.

D. Distinction from Cellular Automata and Deep Learning

Cellular automata often produce localized patterns that repeat but are not logically linked in the way GenesisBoard's 2-cells are. Similarly, deep learning

architectures extract statistical regularities but do not typically generate explicit symbolic equalities. GenesisBoard differs by **constructing and verifying algebraic relationships** during growth, moving beyond emergent shapes into emergent logic.

E. Limitations and Potential Criticisms

A critical limitation is that the observed motifs may be artifacts of the chosen ruleset and seed. If so, the resulting truth density reflects the quirks of a single formal universe rather than an emergent universal property of categorical growth. Testing this requires running ensembles of models with different seeds, rulesets, and even alternative category-theoretic constraints to see which truths persist across worlds.

VI. Implications and Future Work

A. Extending GenesisBoard to Richer Categorical Structures

The current GenesisBoard framework halts at 2-cell formation, meaning it detects and records commutative diagrams but does not build higher-dimensional coherence data such as 3-cells or n-cells. Extending the system to enriched categories, double categories, or infinity-categories would allow the growth process to capture far more complex relationships — potentially revealing patterns analogous to naturality squares, higher homotopies, or adjunction structures. These could be tested for recurrence and stability just as 2-cells are now.

B. Applications in Symbolic AI and Automated Conjecture Generation

GenesisBoard demonstrates that purely symbolic systems can produce **nontrivial verified equalities** without human intervention. This suggests a role as a local-truth discovery engine: running in parallel with theorem provers, it could generate candidate lemmas that mathematicians might not consider otherwise. More ambitiously, such a system could be embedded in symbolic AI pipelines as a

generator of “hypothetical structures” that downstream systems attempt to prove or falsify in general mathematical contexts.

C. Philosophical Impact: Local Truth as a Productive Lens

Philosophically, GenesisBoard reminds us that all human mathematical knowledge is local — bounded by the axioms and inference rules we choose. By watching local truths emerge in a synthetic microcosm, we gain insight into how universes of thought self-organize from minimal premises. This has implications not only for mathematics but also for epistemology, where truth is always situated.

D. Hardware and Performance Considerations

The current experiments were conducted on commodity hardware, with runtimes spanning hours. High-end platforms — multi-core CPUs with large caches, high-memory GPUs, or dedicated FPGA/ASIC designs — could allow orders-of-magnitude more epochs, enabling exploration of deeper attractor basins and more complex categorical landscapes. Larger runs may reveal whether certain motifs appear only at late times, analogous to rare events in statistical mechanics.

E. Next Experimental Steps

1. **Rule Variations:** Introduce stochastic elements into morphism composition rules to test robustness of motif formation.
2. **Cross-Model Testing:** Implement multiple category-theoretic frameworks to see if observed motifs persist.
3. **Conjecture Extraction:** Translate emergent equalities into formal conjectures testable in established proof assistants.
4. **Scaling Studies:** Run on high-performance clusters to push the growth horizon and detect long-term structural plateaus.

VII. Conclusion

The first GenesisBoard paper documented the emergence of coherent categorical structures from a minimal set of compositional rules, showing that nonrandom

motifs such as commutative triangles and multi-path attractors arise spontaneously in a closed symbolic growth process. That work established the factual basis for what we now call *local truths* — equalities and relationships that are verifiably true within the model's own axiomatic boundaries, regardless of their truth in general mathematics.

This second study builds directly on that foundation, shifting the emphasis from *what* the system discovers to *how* and *why* those discoveries matter. By treating the GenesisBoard model as a self-contained mathematical universe, we see that truths generated within it are no less real for being local. Indeed, their locality is precisely what makes them valuable: they reveal the internal geometry and logic of the system's world, much as general relativity describes spacetime curvature within its own domain of applicability.

In this sense, GenesisBoard becomes more than an experimental curiosity. It is a working demonstration that truth can emerge in constrained systems without external guidance, providing a bridge between automated conjecture generation in mathematics, symbolic AI, and the philosophical study of epistemic boundaries.

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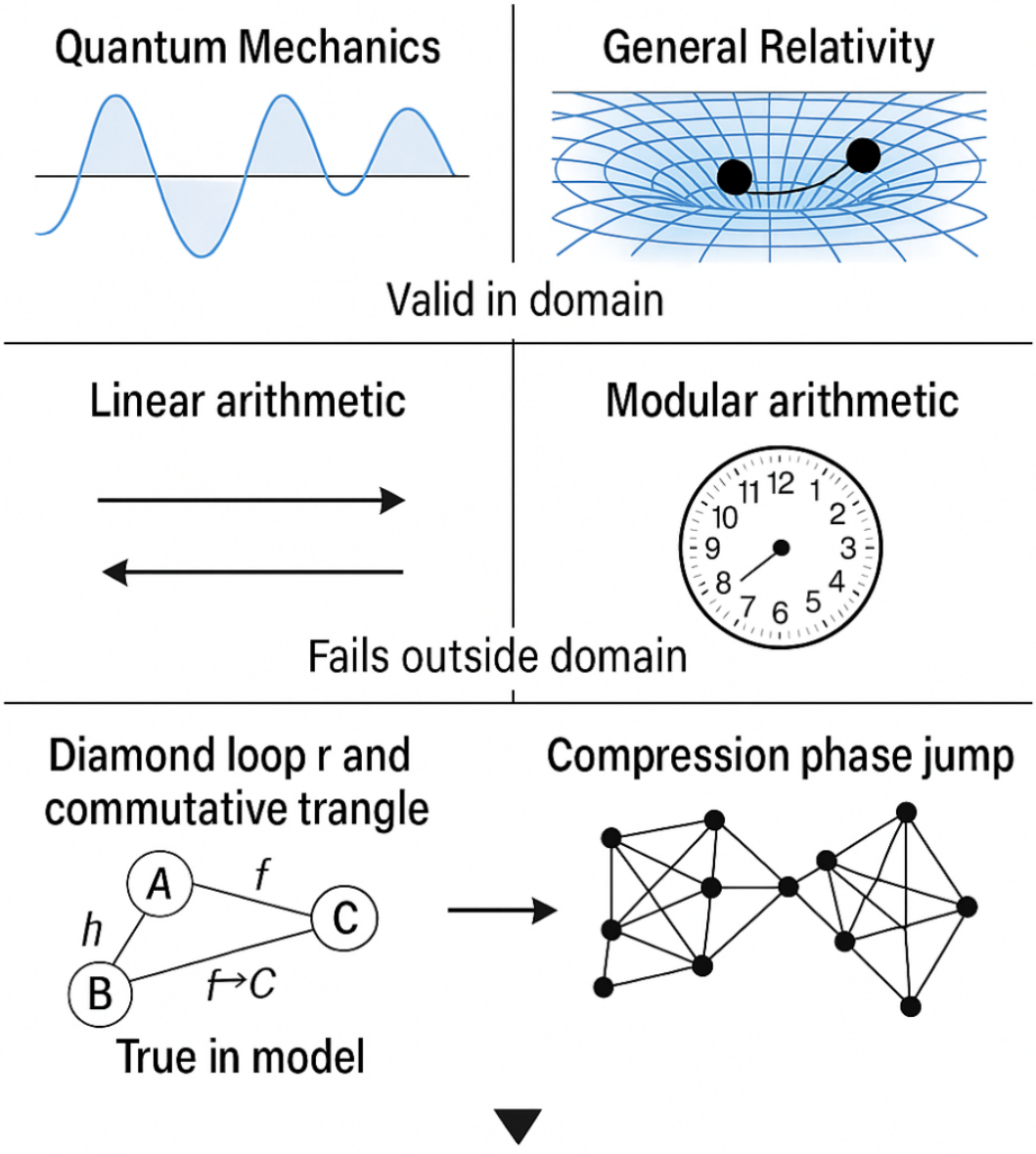
Local truths—whether in physics, mathematics, or categorical simulations—are valid within the axioms and constraints of their domain. GenesisBoard reproduces this pattern, generating truths emergent from its own coherence rules.



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